INSTITUTE - UIE

Bachelor of Engineering (Computer Science & Engineering)

Subject Name : CALCULUS & VECTOR SPACES

Subject Code : 20SMT-175

BE :CSE(All IT branches)

DISCOVER. LEARN. EMPOWER

Introduction to Vector Spaces

Course Outcomes

CO Number	Title	Level
CO1	1) The concept of partial derivatives and its application in real life situations 2) The concept of Multiple Integrals and its applications.	**Remember & Understand**
CO2	The concept of Group theory and its application of analysis to Engineering problems.	**Remember & Understand**
CO3	The concept of vector spaces in a comprehensive manner.	**Remember & Understand**

- Course prerequisites

➢ Basic Knowledge of Sets.

➢ Basic Knowledge of **binary operations.**

➢ Basic Knowledge of functions.

➢ Basic Knowledge of Matrices.

Topic outcomes

- Students will able to understand basic concept of Algebraic Structures like Vector Spaces.
- Students will able to understand basic concept and properties of vector space and subspaces.
- Students now able to aware with techniques of linear algebra.

VECTOR SPACES

- **Definition:**

A **vector space** is a nonempty set V of objects, called *vectors*, on which are defined two operations, called *addition and multiplication by scalars* (real numbers), subject to the ten axioms (or rules) listed below. The axioms must hold for all vectors **u**, **v**, and **w** in V and for all scalars c and d.

1) The sum of **u** and **v**, denoted by $u+v$, is in V.
2) $u+v=v+u$
3) $(u+v)+w=u+(v+w)$
4) There is a zero vector 0 in V such that $u+(-u)=0$

VECTOR SPACES

5) For each **u** in *V*, there is a vector –u in *V* such that u+(-u)=0
6) The scalar multiple of **u** by *c*, denoted by *c***u**, is in *V*
7) c(u+v)=cu+cv
8) (c+d)u=cu+du
9) c(du)=(cd)u
10) 1u=u

Example: The space of all 3x3 matrices is a vector space. The space of all matrices is **not** a vector space.

SUBSPACE

- **Definition:**

A **subspace** of a vector space *V* is a subset *H* of *V* that has three properties:

1) The zero vector of *V* is in *H*.
2) *H* is closed under vector addition. That is, for each **u** and **v** in *H*, the sum is in H.
3) *H* is closed under multiplication by scalars. That is, for each **u** in *H* and each scalar *c*, the vector *c***u** is in *H*.

- Properties (a), (b), and (c) guarantee that a subspace *H* of *V* is itself a *vector space*, under the vector space operations already defined in *V*.
- Every subspace is a vector space.

Example: The space of all 3x3 upper triangular matrices is a subspace. The space of all matrices with integer entries is not.

A SUBSPACE SPANNED BY A SET

- The set consisting of only the zero vector in a vector space V is a subspace of V, called the **zero subspace** and written as $\{\mathbf{0}\}$.

- As the term **linear combination** refers to any sum of scalar multiples of vectors, and Span $\{\mathbf{v}_1,\ldots,\mathbf{v}_p\}$ denotes the set of all vectors that can be written as linear combinations of $\mathbf{v}_1,\ldots,\mathbf{v}_p$.

- **Example:** Given $\mathbf{v}_1$ and $\mathbf{v}_2$ in a vector space V, let H=Span($\mathbf{v}_1$, $\mathbf{v}_2$). Then H is a subspace of V.

A SUBSPACE SPANNED BY A SET

- **Theorem 1:** If $\mathbf{v}_1,\ldots,\mathbf{v}_p$ are in a vector space V, then Span $\{\mathbf{v}_1,\ldots,\mathbf{v}_p\}$ is a subspace of V.

- We call Span $\{\mathbf{v}_1,\ldots,\mathbf{v}_p\}$ **the subspace spanned** (or **generated**) by $\{\mathbf{v}_1,\ldots,\mathbf{v}_p\}$.

- Give any subspace H of V, a **spanning** (or **generating**) set for H is a set $\{\mathbf{v}_1,\ldots,\mathbf{v}_p\}$ in H such that H=Span $\{\mathbf{v}_1,\ldots,\mathbf{v}_p\}$.

Linear Independence and Dependence

- A set of vectors $\{v_1, v_2, \dots, v_k\}$ is ***linearly independent*** if the vector equation

$$x_1 v_1 + x_2 v_2 + \cdots + x_k v_k = 0$$

has only the trivial solution $x_1 = x_2 = \cdots = x_k = 0$.

Example: In set of Reals any two nonzero vectors are linearly dependent.

- A set of vectors $\{v_1, v_2, \dots, v_k\}$ is ***linearly dependent*** if there exist numbers $x_1, x_2, \dots, x_k$, not all equal to zero, such that

$$x_1 v_1 + x_2 v_2 + \cdots + x_k v_k = 0$$

Example: In the space of real polynomials in X, $\{1, t, t^2\}$ are linearly independent.

Basis

Let V be a vector space. A minimal set of vectors in V that spans that spans is called a **basis** for is called a **basis** for V

Equivalently, a basis for V is a set of vectors that

- is linearly independent;
- Spans V.

Or

A set *B* of vectors in a vector space *V* is called a **basis** if every element of *V* may be written in a unique way as a finite linear combination of elements of *B*.

Dimension of a vector space

Let V be a vector space not of infinite dimension. An important result in linear algebra is the following

- Every basis for V has the same number of vectors.
- The number of vectors in a basis for V is called dimension of V, denoted by **dim(V)**.

Example:

1) The dimension of R^n is n.

2) The dimension of the vector space of polynomials in x with real coefficients having degree at most two is 3.

3) A vector space that consists of only the zero vector has dimension zero

References

Reference Books

- Elements of Discrete Mathematics, (Second Edition) C. L. Liu, McGraw Hill, New Delhi, 2017
- Graph Theory with Applications, J. A. Bondy and U. S. R. Murty, Macmillan Press, London.
- Topics in Algebra, I. N. Herstein, John Wiley and Sons. Digital Logic & Computer Design, M. Morris Mano, Pearson.

Online Video Sites:

1. NPTEL
2. Coursera
3. Unacademy

THANK YOU